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Inclusion of Orography in the Problem of the Motion of a Barotropic Atmosphere Over a Spherical Earth

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The problem of nonlinear oscillations of a barotropic atmosphere over an orographically uneven Earth is solved in the case when the orography is represented by a single spherical harmonic.

In recent years, in connection with the increased interest in the problem of blocking, a large number of papers have been published on the question of the interaction of the atmosphere with the underlying surface. An extensive bibliography on this subject is given in [1 -3].

References [4, 5], which have become classics, led to the discovery of a new type of instability of atmospheric flows—orographic instability, physically linked with the nonconservation of the angular momentum of the atmosphere over an orographically uneven Earth. The role of orography in the interaction of Rossby waves with the zonal flow was investigated in [4 -6] in the β-plane approximation. In this paper the sphericity of the Earth is taken into account in the study of the effect of orography on the motion of a barotropic atmosphere. A similar problem, concerning primarily the study of stationary states, was also studied in [7].

1. The equation for the axial component of the angular momentum of a unit mass of air, neglecting friction, has the form

$$\frac{dM}{dt}=-\frac{1}{\rho}\frac{\partial p}{\partial \lambda}. \tag{1}$$

Here $M=(u+a\omega\sin\theta)a\sin\theta$, a is the radius of the Earth, ω is the angular rotational velocity of the Earth, θ is the complement of the latitude with respect to $\pi/2$, λ is the longitude, t is the time, u is the zonal velocity of the wind, p is the pressure, and ρ is the density. Adding (1) to the equation of continuity and integrating over the entire atmosphere, we obtain

$$\frac{d}{dt}\iiint\limits_V M\rho dv=-\iiint\limits_V \frac{\partial p}{\partial \lambda}dv. \tag{2}$$

We transform the volume integral on the right side of (2) into a surface integral by using the properties of integrals which are differentiable with respect to a parameter and we write the equation for the angular momentum in the form

$$\frac{d}{dt}\iiint\limits_V M\rho dv=-\frac{1}{2}\iiint\limits_S\left[p_s\frac{\partial h}{\partial \lambda}-h\frac{\partial p_s}{\partial \lambda}\right]d\sigma, \tag{3}$$

where the function $h(\lambda, \theta)$ describes the relief of the Earth's surface, p_s is the pressure at the surface, and S is the null surface. It is evident from (3) that the axial component of the total angular momentum is not conserved in the presence of orography. The sum of the angular momenta of the Earth and of the atmosphere is conserved.

2. We average the equations describing the dynamics of the atmosphere over altitude with a weight ρ. Assuming that the entropy of air is constant in the entire atmosphere, while the wind velocity is virtually independent of the vertical coordinate z, we obtain a generalization of the equations of the barotropic model of the atmosphere [8] to the case of orography in a spherical coordinate system:

$$\begin{gathered}\frac{\partial\bar u}{\partial t}+\frac{\bar u}{a\sin\theta}\frac{\partial\bar u}{\partial\lambda}+\frac{\bar v}{a}\frac{\partial\bar u}{\partial\theta}+\frac{\bar v\bar u}{a}\operatorname{ctg}\theta=\\ -\frac{c_0^2}{a\sin\theta}\frac{\partial\mu}{\partial\lambda}-\frac{g}{a\sin\theta}\frac{\partial h}{\partial\lambda}-2\omega\cos\theta\bar v,\\ \frac{\partial\bar v}{\partial t}+\frac{\bar u}{a\sin\theta}\frac{\partial\bar v}{\partial\lambda}+\frac{\bar v}{a}\frac{\partial\bar v}{\partial\theta}-\frac{\bar u^2}{a}\operatorname{ctg}\theta=\\ -\frac{c_0^2}{a}\frac{\partial\mu}{\partial\theta}-\frac{g}{a}\frac{\partial h}{\partial\theta}+2\omega\cos\theta\bar u,\\ \frac{\partial\mu}{\partial t}+\frac{1}{a\sin\theta}\left(\frac{\partial\bar u\mu}{\partial\lambda}+\frac{\partial\bar v\mu\sin\theta}{\partial\theta}\right)=0,\\ (\bar u,\bar v)=\int\limits_h^\infty(u,v)\rho dz\Big/\int\limits_h^\infty\rho dz,\ \mu=p(0,x,y,t)/p_{00},\ c_0^2=RT_{00},\end{gathered} \tag{4}$$

p_{00} = 1000 mbar and T_{00} is the average air temperature at the surface. The system (4) conserves potential vorticity and energy

$$\frac{d}{dt}\left[\left(\frac{1}{a\sin\theta}\frac{\partial\bar{u}\sin\theta}{\partial\theta}-\frac{1}{a\sin\theta}\frac{\partial\bar{v}}{\partial\lambda}+2\omega\cos\theta\right)\Big/\mu\right]=0, \quad (5)$$

$$\frac{d}{dt}\iint_S\left[\mu\left(\frac{\bar{u}^2+\bar{v}^2}{2}\right)+\frac{c_0^3\mu^2}{2}+\mu gh\right]d\sigma=0. \quad (6)$$

3. The law of conservation of the potential vorticity (5) in the quasigeostrophic approximation is written in the form (see [8])

$$\frac{\partial}{\partial t}q+[\psi,q]=0,\ q=\Delta\psi+l+\xi-\frac{1}{L_0^2}\psi, \quad (7)$$

where $\psi=(c_0^2/l_{av})\mu+(g/l_{av})h$, $L_0^2=c_0^2/l_{av}^2$, $H=c_0^2/g$ is the scale height of the atmosphere, l_{av} is the average value of the Coriolis parameter, and g is the acceleration of gravity. In the derivation of (7) it was assumed that the ratio of the characteristic height of the relief to the scale height of the atmosphere h/H is of the same order of magnitude as the Kibel number $Ki\approx 0.1$. In what follows we shall neglect the term describing the two-dimensional compressibility in (7) because of its smallness. Separating the zonal component of the flow and the deviation from zonality and neglecting the effect of the interaction of the nonzonal components on the nonzonal part of the flow, we arrive at the following system of equations (the wavy overbar denotes zonally averaged quantities, while the prime denotes the deviation from the zonally averaged quantities):

$$\frac{\partial\tilde{q}}{\partial t}-\frac{1}{a^2\sin\theta}\frac{\partial}{\partial\theta}\widetilde{\left(\frac{\partial\psi'}{\partial\lambda}q'\right)}=0,$$
$$\frac{\partial q'}{\partial t}+\frac{1}{a^2\sin\theta}\left(\frac{\partial\tilde{\psi}}{\partial\theta}\frac{\partial q'}{\partial\lambda}-\frac{\partial\psi'}{\partial\lambda}\frac{\partial\tilde{q}}{\partial\theta}\right)=0, \quad (8)$$
$$\tilde{q}=\frac{1}{a^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\tilde{\psi}}{\partial\theta}\right)+2\omega\cos\theta,\ q'=\Delta\psi'+\xi.$$

Multiplying the first equation in (8) by sin θ and integrating first over θ from zero to θ (assuming that the flux of the vortex at the pole equals zero) and then over the entire Earth's surface, we obtain for this model the equation for the change in the relative angular momentum

$$\frac{\partial}{\partial t}M_{rel}=\frac{1}{2}\iint_S\left(\frac{\partial\psi'}{\partial\lambda}\xi-\psi'\frac{\partial\xi}{\partial\lambda}\right)d\sigma\Big/\iint_S d\sigma=\frac{1}{2}\overline{\overline{\left(\frac{\partial\psi'}{\partial\lambda}\xi-\psi'\frac{\partial\xi}{\partial\lambda}\right)}},$$
$$M_{rel}=\iint_S\sin\theta\frac{\partial\tilde{\psi}}{\partial\theta}d\sigma\Big/\iint_S d\sigma=I\omega\alpha,\ I={}^2/_3a^2, \quad (9)$$

where α is the circulation index.

For a zonal flow of the form $\tilde{\psi}=-\alpha(t)\omega a^2P_1^0(\theta)+\beta(t)P_n^0(\theta)$ from the system (8) it is possible to derive a conservation law which extends the integral of motion found in [9] (for the barotropic vorticity equation linearized with respect to the indicated zonal flow) to the case when orography is taken into account

$$\frac{\overline{\overline{(q')^2}}}{2}-n(n+1)\frac{\overline{\overline{(\nabla\psi')^2}}}{a^2 2}=\text{const.} \quad (10)$$

Multiplying the second equation of the system (8) by ψ' and q' in turn and integrating over the entire Earth's surface we obtain equations which describe the change in the kinetic energy of the atmosphere and of the average squared nonzonal component of the vorticity, combining which with (9) we arrive at the equation

$$\frac{\partial}{\partial t}\left\{\frac{\overline{\overline{(\nabla\psi')^2}}}{2}-\frac{a^2}{n(n+1)}\frac{\overline{\overline{(q')^2}}}{2}+\frac{I(\alpha\omega)^2}{2}\left[1-\frac{2}{n(n+1)}\right]-\frac{2\omega}{n(n+1)}I\omega\alpha\right\}=0. \quad (11)$$

Thus, taking into account the orography owing to the nonconservation of the angular momentum, leads in the law of conservation for the quasilinear system (8) to the appearance of additional terms as compared with the analogous formula for a smooth Earth [9].

Here and everywhere below the changes in ω caused by exchange of angular momenta between the atmosphere and the solid Earth are neglected, since the moment of inertia of the atmosphere is vanishingly small compared with that of the Earth (their ratio $I_a/I_3\sim 10^{-8}$). For this reason the solid Earth can be regarded as an infinite reservoir of angular momentum (Ω-stat) analogously to the manner in which the thermostat in thermodynamics is viewed as an infinite reservoir of thermal energy.

In the case when the angular velocity of the zonal flow is constant over θ, i.e., $\tilde{\psi}=\alpha(t)a^2\omega P_1^0$, the integral (11) separates into two independent integrals:

$$\frac{\partial}{\partial t}\left[\frac{\overline{\overline{(\nabla\psi')^2}}}{2}+\frac{1}{2}I(\alpha\omega)^2\right]=0, \quad (12)$$

$$\frac{\partial}{\partial t}\left[\frac{\overline{\overline{(q')^2}}}{2}+\frac{2}{a^2}I\frac{(\alpha\omega)^2}{2}+\frac{2}{a^2}\omega I\omega\alpha\right]=0. \quad (13)$$

The first integral expresses the law of conservation of energy, while the second expresses the law of conservation of enstrophy; in addition, the second integral of motion implies that some combination of the enstrophy of the nonzonal flow and the relative angular momentum is conserved approximately (the quantity $2I(\alpha\omega)^2/2a^2$ is much smaller than the two other terms in (13)). Therefore, reducing $(q')^2$ to zero increases M to a maximum value. From the relation

$$\frac{\overline{\overline{(q')^2}}}{2}+\frac{2}{a^2}\frac{I(\alpha\omega)^2}{2}+\frac{2\omega}{a^2}I\omega\alpha=\frac{2}{a^2}I\frac{(\alpha_{max}\omega)^2}{2}+\frac{2\omega}{a^2}I\omega\alpha_{max}$$

we obtain

$$I\omega(\alpha_{max}-\alpha)=\frac{\overline{\overline{(q')^2}}/2}{\frac{2\omega}{a^2}\left(1+\frac{\alpha_{max}+\alpha}{2}\right)}. \quad (14)$$

Starting from this formula we can introduce the concept of a deficit of angular momentum. This is the maximum quantity of angular momentum which can be transferred from the solid Earth to the atmosphere owing to the interaction of Rossby waves with the orography.

4. The expression for the deficit of angular momentum can also be obtained directly from the nonlinear equation (7). The formula for the absolute angular momentum in the barotropic quasi-geostrophic model of the atmosphere is written in the form

$$M = \frac{p_{00}}{g} \iint\limits_S \left[\frac{1}{a}\frac{\partial\psi}{\partial\theta} + a\omega\sin\theta\right] a\sin\theta d\sigma. \qquad (15)$$

Integrating this equation by parts we obtain

$$M = \frac{p_{00}}{g} \iint\limits_S q\cos\theta a^2\, d\sigma. \qquad (16)$$

For (7) there exists an integral conservation law for the distribution of the vorticity

$$\frac{d}{dt}\iint\limits_S \Phi(q)\, d\sigma = \frac{dE}{dt} = 0$$

(Φ is an arbitrary function). We shall calculate the extremum M under the condition F = const. For this we find the absolute extremum of the functional $H = M + F$. The conditions of the extremum will be $\delta H = 0$ and $\delta^2 H < 0$ or $\delta^2 H > 0$. The equality

$$\delta H = \iint\limits_S \left(\frac{p_{00}}{g}\cos\theta a^3 + \Phi'(q)\right)\delta q d\sigma = 0$$

implies that q must depend only on θ, i.e., the vorticity distribution must be strictly zonal. The expression for the second variation

$$\delta^2 H = \iint\limits_S \frac{\Phi''(q)}{2}(\delta q)^2\, d\sigma = \frac{1}{2}\iint\limits_S \frac{a^3\sin\theta}{\partial q/\partial\theta}(\delta q)^2\, d\sigma \qquad (17)$$

implies that M will have an extremal value, if $\partial q/\partial\theta$ is a monotonic function. In the case of a zonal flow rotating with a θ-independent angular velocity $\alpha\omega$, $\delta^2 H < 0$ and M assumes its maximum value; the formula (17), taken with the opposite sign, is identical to (14) and is thus the most general expression for the deficit of angular momentum.

5. We shall now solve the system (8). Defining the relief of the Earth's surface by the function

$$\xi = \sum_m (C_n^m \sin m\lambda + D_n^m \cos m\lambda) P_n^m(\theta), \qquad (18)$$

we seek the solution of (8) in the form

$$\psi = -\alpha(t)\omega a^2\cos\theta + \sum_m (A_n^m(t)\sin m\lambda + B_n^m(t)\cos m\lambda) P_n^m(\theta). \qquad (19)$$

Substituting (18) and (19) into (8), we obtain the following system of equations:

$$\begin{gathered} I\omega\frac{d\alpha}{dt} = \frac{1}{2}\sum_m (A_n^m D_n^m - C_n^m B_n^m) m N_n^m, \\ \frac{dB_n^m}{dt} = -m\omega\varepsilon_n A_n^m + \frac{\omega a^2\alpha m C_n^m}{n(n+1)}, \\ \frac{dA_n^m}{dt} = m\omega\varepsilon_n B_n^m - \frac{\omega a^2 \alpha m D_n^m}{n(n+1)}, \end{gathered} \qquad (20)$$

where

$$N_n^m = \int_0^\pi [P_n^m(\theta)]^2\sin\theta d\theta,\quad \varepsilon_n = \alpha - \frac{2(1+\alpha)}{n(n+1)}.$$

For this system the integrals of the motion (12) and (13) assume the form, respectively,

$$\frac{d}{dt}\sum_m \frac{(A_n^m)^2 + (B_n^m)^2}{4} N_n^m \frac{n(n+1)}{a^2} + \frac{I\omega^2\alpha^2}{2} = \frac{dE}{dt} = 0, \qquad (21)$$

$$\frac{d}{dt}\left[I\omega^2\int_0^\alpha \varepsilon_n(\alpha')\,d\alpha' + \sum_m N_n^m \frac{1}{4}(B_n^m D_n^m + A_n^m C_n^m)\right] = \frac{dF}{dt} = 0. \qquad (22)$$

An extremum of E with F = const is a sufficient condition for the stability of the stationary solution of the nonlinear system (20). To find the conditional extremum of E we form the functional $H = E + kF$ (k is an undetermined Lagrange multiplier) and we find its absolute extremum, which exists, if

$$\begin{gathered} \delta H = \sum_m \left[\left(\frac{A_n^m}{2}\frac{n(n+1)}{a^2} + \frac{k}{4}C_n^m\right) N_n^m \delta A_n^m + \right. \\ \left. + \left(\frac{B_n^m}{2}\frac{n(n+1)}{a^2} + \frac{k}{4}D_n^m\right) N_n^m\delta B_n^m\right] + \left(\alpha + \frac{k}{2}\varepsilon_n\right) I\omega^2\delta\alpha = 0, \end{gathered} \qquad (23)$$

while the quadratic form

$$\delta^2 H = \sum_m \frac{(\delta A)_n^m)^2 + (\delta B_n^m)^2}{4} N_n^m \frac{n(n+1)}{a^2} - \frac{I\omega^3(\delta\alpha)^2}{\varepsilon_n n(n+1)}$$

has a definite sign. The stationary solution of the system (20) satisfies the condition (23) and, according to (24), is stable for all $\varepsilon_n > 0$.

6. In the first approximation the system (20) reduces to a dynamic system of third order for the coefficients A, B, and α, which can be integrated explicitly because of the existence of the first two integrals of motion. In dimensionless variables the system is written in the form (see also [10])

$$\begin{gathered} \frac{d\hat{A}}{d\hat{t}} - m\varepsilon_n\hat{B} = 0,\quad \frac{d\hat{B}}{d\hat{t}} + m\varepsilon_n\hat{A} = \frac{\alpha m\hat{C}}{n(n+1)}, \\ \frac{d\alpha}{d\hat{t}} + \frac{1}{2}jmN_n^m\hat{C}\hat{B} = 0. \end{gathered} \qquad (25)$$

Here $\hat{A} = A_n^m(a^2\omega)$, $\hat{B} = B_n^m/(a^2\omega)$, $\varepsilon_n = \alpha - 2(1+\alpha)/n(n+1)$, $\hat{t} = t\omega$, $\hat{C} = C_n^m/\omega$, $j = {}^3/_2$ (we omit below the caret over the letters). The stationary solution of the system (25) is given by

$\alpha_{st}=\text{const},\ B_{st}=0,\ A_{st}=\alpha_{st}\,C/[\alpha_{st}\,[n(n+1)-2]-2].$

We shall study the stability of the system (25) linearized with respect to this solution. Substituting $\alpha_{CT}+\alpha'$, B', $A_{CT}+A'$ for α, B, and A and neglecting terms quadratic in α', B', and A', we obtain

$$\frac{d^2B'}{dt^2}+m^2\left[\varepsilon^2_{n\,st}-\frac{jN^m_nC^2}{n^2(n+1)^2\varepsilon_{n\,st}}\right]B'=0. \qquad (26)$$

From here the stationary solution of the system will be unstable for

$$\sqrt[3]{\frac{jNC^2}{n^3(n+1)^2}}\geqslant\varepsilon_{n\,st}\geqslant 0. \qquad (27)$$

Equation (26) also implies that the highest rate of growth of the wave is achieved in the limit $\varepsilon_{n\,st}\to 0$, i.e., when the phase velocity of the Rossby wave approaches zero.

With the help of the integrals of motion

$$E=\left[\frac{A^2+B^2}{4}+\frac{\alpha^2}{2jN^m_n n(n+1)}\right], \qquad (28)$$

$$F=\left[\int\limits_0^{\alpha}\varepsilon_n(\alpha')\,d\alpha'+\frac{ACN^m_n j}{2}\right] \qquad (29)$$

the nonlinear system (25) can be reduced to the equation

$$\frac{d\alpha}{dt}=\frac{m[n(n+1)-2]}{2n(n+1)}\left\{\frac{(Ej^2N^{m2}_nC^2-F^2)\,4n^2(n+1)^2}{[n(n+1)-2]^2}-\alpha^4+\frac{8}{[n(n+1)-2]}\alpha^3-\left\{\frac{16}{[n(n+1)-2]^2}+\frac{2jN^m_nC^2n(n+1)}{[n(n+1)-2]^2}-\frac{4Fn(n+1)}{n(n+1)-2}\right\}\alpha^2-16F\frac{n(n+1)}{[n(n+1)-2]^2}\alpha\right\}^{1/2}, \qquad (30)$$

which can be integrated in terms of elliptic functions [11] (the values of E and F are determined from the initial conditions). Since the zonal flux is a positive quantity, the limits of variation of α are determined from the condition that the expression in the radicand in (30) is positive. The coefficient in the highest power of the polynomial is negative, and therefore solutions corresponding to the cases of four real roots or two real and two complex roots are physically meaningful. For the case when all four roots are real, the solution of Eq. (30) is written in the form

$$\alpha=\frac{l_{i+1}\,\mathrm{sn}^2([\delta t+\varphi_i],\tilde{k})-(l_{i-1}-l_{i+1})\,l_i/(l_{i-1}-l_i)}{\mathrm{sn}^2([\delta t+\varphi_i],\tilde{k})-(l_{i-1}-l_{i+1})/(l_{i-1}-l_i)}, \qquad (31)$$

where $l_1>l_2>l_3>l_4$ are the roots of the polynomial, $i=4,\ 2,\ l_5=l_1,\ \tilde{k}=\sqrt{(l_1-l_2)(l_3-l_4)/(l_1-l_3)(l_2-l_4)}$, and

$$\delta=\frac{\sqrt{(l_1-l_2)(l_3-l_1)}\,m\,[n(n+1)-2]}{4n(n+1)},$$

$$\varphi_i=\frac{\sqrt{(l_4-l_2)(l_3-l_1)}}{2}\int\limits_{l_i}^{\alpha_0}\frac{d\alpha}{\sqrt{-(\alpha-l_1)(\alpha-l_2)(\alpha-l_3)(\alpha-l_4)}}.$$

For the case when two roots are real and two are complex,

$$\alpha=\frac{4(pm_2-qm_1)\,\mathrm{sn}^2([\delta t+\varphi],\tilde{k})\,\mathrm{cn}^2([\delta t+\varphi],\tilde{k})-pm_2}{4(p-q)\,\mathrm{sn}^2([\delta t+\varphi],\tilde{k})\,\mathrm{cn}^2([\delta t+\varphi],\tilde{k})-p}, \qquad (32)$$

$m_1>m_2$, $m_3=c+id$, $m_4=c-id$ are the roots of the polynomial, $p^2=(c-m_1)^2+d^2$,

$$q^2=(c-m_2)^2+d^2,\ \delta=\sqrt{pq}\,\frac{m[n(n+1)-2]}{2n(n+1)},$$

$$\tilde{k}=\frac{1}{2}\sqrt{\frac{-(p-q)^2+(m_1-m_2)^2}{pq}},$$

$$\varphi=\sqrt{pq}\int\limits_{\alpha}^{\alpha_0}\frac{d\alpha}{\sqrt{-(\alpha-m_1)(\alpha-m_2)[(\alpha-c)^2+d^2]}}.$$

The period of the oscillations T of the circulation index α is given, respectively, by the formula

$$T=\frac{4n(n+1)}{m[n(n+1)-2]}\int\limits_{l_i}^{l_{i+1}}\frac{d\alpha}{\sqrt{-(\alpha-l_1)(\alpha-l_2)(\alpha-l_3)(\alpha-l_4)}}=\frac{8n(n+1)\,F(\pi/2,\sqrt{(l_1-l_2)(l_3-l_4)/(l_1-l_3)(l_2-l_4)})}{m[n(n+1)-2]\sqrt{(l_4-l_2)(l_3-l_1)}} \qquad (33)$$

for the solution (31) and by the formula

$$T=\frac{8n(n+1)}{m[n(n+1)-2]\sqrt{pq}}F\left(\pi/2,\frac{1}{2}\sqrt{\frac{-(p-q)^2+(m_1-m_2)^2}{pq}}\right) \qquad (34)$$

for the solution (32) ($F(\pi/2,k)$ is the complete elliptic integral of the first kind).

Figure 1 shows the dependence of the period of the oscillations of the cycle of the index α

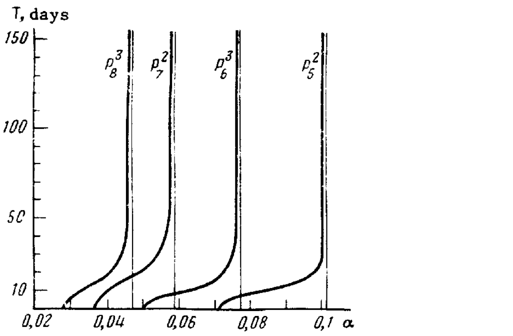

Fig. 1. Dependence of the period of the oscillations of the cycle of the index on the initial value of α_0 for the case $B\to 0$ and $A\to A_{st}=\alpha_0C/\alpha_0[n(n+1)-2]-2$.

Table 1

		α	H			
			0,03	0,06	0,09	0,12
$\xi = C_7^2 \sin 2\lambda\, P_7^2(\theta)$, $A_0 = -0{,}0002$, $B_0 = 0$	T, days	0,02	32,8	23,6	18,3	15,1
	Δα		0,027—0,02	0,024—0,02	0,0203—0,02	0,02—0,017
	T, days	0,03	38,1	24,0	18,2	15,0
	Δα		0,03—0,0287	0,03—0,021	0,03—0,016	0,03—0,011
	T, days	0,04	37,7	23,9	18,2	14,9
	Δα		0,04—0,023	0,04—0,015	0,04—0,009	0,04—0,005
	T, days	0,045	38,1	24,0	18,2	14,9
	Δα		0,045—0,018	0,045—0,012	0,045—0,006	0,045—0,001
$\xi = C_8^3 \sin 3\lambda\, P_8^3(\theta)$, $A_0 = -0{,}00002$, $B_0 = 0$	T, days	0,02	30,1	18,6	14,1	11,5
	Δα		0,025—0,02	0,02—0,0199	0,02—0,016	0,02—0,012
	T, days	0,03	28,1	18,0	13,7	11,3
	Δα		0,03—0,019	0,03—0,013	0,03—0,009	0,03—0,005
	T, days	0,04	28,2	18,1	13,7	11,2
	Δα		0,04—0,01	0,04—0,005	0,04—0,0006	0,04—-0,003
	T, days	0,045	30,8	18,4	13,8	11,3
	Δα		0,045—0,006	0,045—0,0006	0,045—-0,004	0,045—-0,007

on the initial value of α_0 for the limiting case when the initial values of the wave components approach the values corresponding to the stationary solution: $B_0 \to 0$, $A_0 \to A_{st} = \alpha_0 C/[\alpha_0[n(n+1)-2]-2]$. Four variants of the function describing the relief were studied: $Y_5^2 = C_5^2 \sin 2\lambda\, P_5^2$, $Y_6^3 = C_6^3 \sin 3\lambda\, P_6^3$, $Y_7^2 = C_7^2 \sin 2\lambda\, P_7^2$, $Y_8^3 = C_8^3 \sin 3\lambda\, P_8^3$. The average height of the mountain

$$H_m = \left(\iint_S (C_n^m P_n^m \sin m\lambda)^2\, d\sigma\right)^{1/2} \Big/ \sqrt{2\pi a^2}$$

is assumed to equal 0.1, which agrees with the data on the expansion of the orography in spherical harmonics and corresponds in dimensional units to a mountain 1.5 km high. It is evident from Fig. 1 that oscillations of α arise only when α_0 falls in the region of instability (27). This region (at the latitude 45°) corresponds to zonal winds 23.5 m/sec $< U_0 <$ 33.3 m/sec for the wave Y_5^2, 16.5 m/sec $< U_0 <$ 25.4 m/sec for Y_6^3, 12.2 $< U_0 <$ 19.4 m/sec for Y_7^2 and 9.4 m/sec $< U_0 <$ 15/5 m/sec for Y_8^3. As the wave number decreases, this region increases, and the subregion corresponding to oscillations with the longest period becomes narrower. For small but nonzero values of B and $A' = A - A_{st}$ the left side of the graph of the dependence of the period on α begins not at zero but rather at some small quantity, while the right side will be not the asymptote going to infinity but rather will approach a large but finite value.

Table 1 gives the results of calculations of the amplitude and period of oscillations of the cycle of the index, carried out using the formulas (31) - (34) for a series of values of H_m, α_0, A_0, B_0 corresponding to the observed values. The orography in these calculations was represented by spherical harmonics Y_7^2, and Y_8^3, for which an oscillatory regime is possible for existing zonal flows. It is evident from the table that oscillations of α with large amplitude with periods longer than one month are possible. We note that similar oscillations of the cycle of the index were obtained in the numerical model of [12] when terms describing the orography were included in the model. Thus, in this work, by simplifying as much as possible the equations of motion of a barotropic atmosphere over an orographically uneven spherical Earth, the interaction of the Rossby wave with the relief, leading to long-period oscillations of the angular momentum, was separated out in a pure form.

In recent years there have appeared works [13, 14] in which a dependence is established between variations of the Earth's angular rotational velocity, measured with high degree of accuracy by astronomical methods, and variations of the angular momentum of the atmosphere. This opens up the possibility of using a quite long series of measurements of the duration of the day in diagnostic investigations of blocking. For this reason, the study of the exchange of angular momentum between the atmosphere and an uneven Earth may be useful in such studies.

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