

# CALCULATION OF THE FIELD OF HEAT INFLUXES BY THE EQUATION OF TRANSFORMATION OF A POTENTIAL VORTEX

G. M. Agayan, M. V. Kurganskii, and I. A. Pisnichenko

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A method is proposed for diagnostic calculation of the field of heat influxes on isentropic surfaces using the equation of transformation of the potential vortex. The method is tested on a limited selection from the FGGE data bank related to the situation of Atlantic blocking in December 1978. Physically valid values of the intensity of "heat sources" in the upper and middle troposphere are obtained, and a certain geographic localization of these sources, which corresponds to the circulation type, is identified.

1. The quality of the parametrization of nonadiabatic factors, i.e., heat influxes and friction, is extremely important for a successful medium-term or long-term weather forecast. Despite great successes achieved here recently, the corresponding problem is far from being solved, especially in describing heat influxes (meaning radial heat influxes and turbulent influxes of explicit and latent heat). Therefore an important role continues to be played by diagnostic methods of calculating heat influxes with respect to known meteorological fields, which anticipates comparison of the obtained values with those which come from the "physics" of prognostic models.

One of the methods of diagnostic calculation of heat influxes, defined as a "discrepancy" in the equation of heat influx, where the remaining terms in this equation are calculated according to factual meteorological data, was proposed by G. P. Kurbatkin [3]. Investigations which led in this direction played a definite role in the improvement of the weather forecast and facilitated the formation of the scientific basis of the Razrezy program [2].

It is convenient to note that the nonadiabatic factors find their most capacious expression in examination of the equations of transformation of adiabatic invariants:

a) entropy  $s$

$$ds/dt = Q/T$$

( $Q$  represents heat influxes calculated for a unit mass,  $T$  is temperature, and  $t$  is time) and

b) potential vortex of Ertel  $I = (\bar{\omega}_a \nabla s)/\rho$  [6,10]

$$\frac{dI}{dt} = \frac{\bar{\omega}_a \nabla (Q/T)}{\rho} + \frac{\text{rot } \bar{F} \nabla s}{\rho}$$

Here  $\bar{\omega}_a$  is the absolute vorticity,  $\bar{F}$  is the nonpotential forces, including the force of friction,  $\rho$  is density, and  $\nabla$  is the gradient operator.

These equations take a simpler form in the isentropic system of coordinates under the assumption of the quasi-static nature of the processes. It deserves attention that the equation of transformation of the potential vortex may be preferable for diagnostic calculation of the field of heat influxes to the equation for entropy if the dissipative factors are parametrized reliably or we assume (as assumed for the free atmosphere) that the dissipative factors play a smaller role than the heat influxes. The proposed method lets us avoid calculating the field of vertical velocities (or their analogs in an isobaric coordinate system) at the intermediate stage, which is an independent problem. In this sense the advantage of isentropic coordinates is the fact that the field of vertical velocity is not explicitly present here, and its role is played by the sought factor itself, the velocity of heat influxes. The implementation of this theoretical scheme is the subject of this note.

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2. As the original equations we will use the equation of motion in  $\Theta$  coordinates in the Gromeki-Lamb form:

$$\frac{\partial u}{\partial t} = -\frac{\partial B}{\partial x} + \omega_{a\theta} v - \dot{\Theta} \frac{\partial u}{\partial \Theta} + F, \quad (1)$$

$$\frac{\partial v}{\partial t} = -\frac{\partial B}{\partial y} - \omega_{a\theta} u - \dot{\Theta} \frac{\partial v}{\partial \Theta} + G, \quad (2)$$

$$B = \frac{u^2 + v^2}{2} + M, \quad (3)$$

$$\frac{\partial M}{\partial \Theta} = \Pi, \quad (4)$$

$$\omega_{a\theta} = l + \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \quad (5)$$

Here  $u$  and  $v$  are projections of the horizontal wind speed  $\bar{V}$  to the Cartesian  $x$  and  $y$  axes with traditional orientation ( $x$  axis is directed toward the east,  $y$  axis to the north),  $B$  is the Bernoulli function,  $M = c_p T + gz = \Pi\Theta + gz$  is the Montgomery flux function,  $\Pi = c_p (p/p_{000})^{R/c_p}$  is the Exner function,  $p_{00} = 1000$  mbar,  $\omega_{a\theta}$  is the absolute vorticity, and  $F$  and  $G$  are the horizontal components of the nonpotential force, including viscosity.

Eliminating the Bernoulli function  $B$  from equations (1) and (2), we quickly come to the equation of conservation of vortex filling (see [11,5])

$$\frac{\partial}{\partial t} \omega_{a\theta} + \frac{\partial}{\partial x} (u \omega_{a\theta} + \dot{\Theta} \frac{\partial v}{\partial \Theta} - G) + \frac{\partial}{\partial y} (v \omega_{a\theta} - \dot{\Theta} \frac{\partial u}{\partial \Theta} + F) = 0. \quad (6)$$

The law of mass conservation in  $\Theta$  coordinates is written in the form

$$\frac{\partial}{\partial t} \rho_{\theta} + \frac{\partial}{\partial x} (u \rho_{\theta}) + \frac{\partial}{\partial y} (v \rho_{\theta}) + \frac{\partial}{\partial \Theta} (\dot{\Theta} \rho_{\theta}) = 0, \quad (7)$$

where  $\rho_{\theta} = -g^{-1} \partial p / \partial \Theta$  plays the role of density in  $\Theta$  coordinates.

From Eqs. (6) and (7) it follows that in the presence of diabatic factors ( $\dot{\Theta} \neq 0$ ) the isentropic surfaces  $\Theta = \text{const}$  play the role of a "semipermeable membrane": the vortex filling, found about the fixed isentropic surface, remains constant everywhere, whereas the corresponding air mass may vary. Since the potential vortex  $q = \omega_{a\theta}$  has the sense of density or specific concentration of the vortex filling, under conditions of local heating, when the air mass included between the isentropic surfaces over the heating region increases, there is a decrease in the concentration of the vortex filling, otherwise, anticyclogenesis. On the other hand, beneath the heating region there is a convergence of isentropic surfaces and as a consequence cyclogenesis (see Fig. 1). Under conditions of local cooling the picture is the opposite. This symmetry is conditioned by the requirements related to the law of conservation of momentum, since no internal cause may lead to the change in the total momentum of the system. In the presence of external (Ekman) friction this symmetry vanishes, and the heating in the surface layer of the atmosphere against the background of dissipation leads to anticyclogenesis, whereas cooling leads to cyclogenesis [1]. This conclusion agrees as a whole with the general energy considerations, since the generation of energy of atmospheric movements is described by the integral

$$\iiint \dot{\Theta} \Pi \rho_{\theta} dx dy d\Theta,$$

from which it is clear that on the average there should be a positive correlation between the field of heat influxes  $\dot{\Theta}$  and the field of the Exner function  $\Pi$  at the isentropic surface. A noteworthy feature of isentropic analysis is the unambiguous relationship of the fields of pressure and temperature at the isentropic surface, since on one hand, because of the formula  $\Pi\Theta = c_p T$  the function  $\Pi$  is proportional to temperature, and on the other hand, because of the definition of the Exner function

$$\Pi = c_p (p/p_{00})^{R/c_p}$$

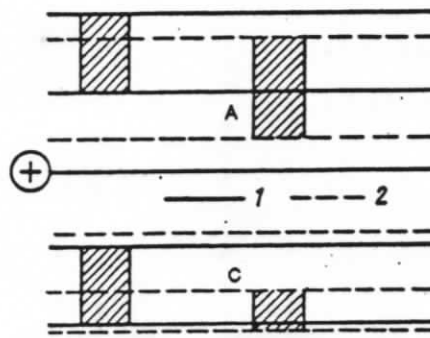


Fig. 1. Diagram illustrating mechanism of anticyclogenesis (A) in region over heat source ( $\oplus$ ) and cyclogenesis (C) in region under heat source due to mechanism of a decrease (increase) in the concentration of the vortex charge in the layer between isentropic surfaces. 1) Isentropic surfaces at initial moment of time  $t = 0$ , 2) at some moment of time  $t = \tau > 0$ .

for air it is proportional to pressure to the power  $2/7$ . Therefore it is clear that at the isentropic surface the anticyclone is simultaneously represented by a positive temperature anomaly, and the cyclone is represented by a negative temperature anomaly. Consequently, to maintain a stationary regime the air should be heated in anticyclones and cooled in cyclones.

By some transformation of equations (6) and (7) and eliminating from them the horizontal divergence of velocity  $\partial u/\partial x + \partial v/\partial y$  we can arrive at the equation of transformation of the potential vortex

$$\rho_{\theta} \frac{d q}{d t} = \omega_{\theta\theta} \frac{\partial \dot{\theta}}{\partial \theta} - \frac{\partial \dot{\theta}}{\partial x} \frac{\partial v}{\partial \theta} + \frac{\partial \dot{\theta}}{\partial y} \frac{\partial u}{\partial \theta} + \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y}, \quad (8)$$

where  $d/dt = \partial/\partial t = u\partial/\partial x + v\partial/\partial y + \dot{\theta}\partial/\partial\theta$  is the symbol of the full derivative.

Now we must simplify the general equation (8). Assuming the movements to be quasi-geostrophic, we neglect the term  $-(\partial\theta/\partial x)(\partial v/\partial\theta) + (\partial\dot{\theta}/\partial y)(\partial u/\partial\theta)$  in the right side of (8) compared to  $\omega_{\theta\theta}\partial\dot{\theta}/\partial\theta$ . Considering that

$$d/dt = d_{\theta}/dt + \dot{\theta}\partial/\partial\theta,$$

where  $d_{\theta}/dt$  is the symbol of the full derivative with movement along the isentropic surface, we can write

$$\rho_{\theta} \frac{d_{\theta} q}{d t} = \rho_{\theta} q^2 \frac{\partial}{\partial \theta} (q^{-1} \dot{\theta}) + \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y} \quad (9)$$

or, equivalently,

$$\frac{d_{\theta}}{d t} (q^{-1}) = - \frac{\partial}{\partial \theta} (q^{-1} \dot{\theta}) - \frac{1}{\rho_{\theta} q^2} \left( \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y} \right). \quad (10)$$

Equation (8) can be rewritten in the form

$$\rho_{\theta} \frac{d_{\theta} q}{d t} = q \frac{\partial}{\partial \theta} (\rho_{\theta} \dot{\theta}) - \frac{\partial}{\partial x} \left( \dot{\theta} \frac{\partial v}{\partial \theta} \right) + \frac{\partial}{\partial y} \left( \dot{\theta} \frac{\partial u}{\partial \theta} \right) + \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y},$$

indicated in [11]. The first term in the right side of the equation, the main one with respect to order, differs from the first term in the right side of equation (9) by the term  $\dot{\theta}\partial\omega_{\theta\theta}/\partial\theta$ , as quite correctly noted in [11]. Because of this we must indicate the inaccuracy on p. 793 of [5] in discussing the results of [11].

3. The main difficulty in the application of (10) for diagnostic investigations is that proceeding from the field of potential vortex on the basis of this equation we cannot simultaneously calculate the heat influxes and the dissipative factors. We therefore assume that in the free atmosphere dissipation can be neglected, i.e., the transformation of the field of potential vortex is completely conditioned by heat influxes  $\Theta$ . In this case we have

$$\frac{d_0}{dt} (q^{-1}) \approx - \frac{\partial}{\partial \Theta} (q^{-1} \dot{\Theta}).$$

Following [5,7], we shift from the invariant  $q$  to the invariant  $\bar{\Omega} = \rho \delta q$ , where  $\rho \delta = -g^{-1} dp^*/d\Theta$ , and  $p^*$  is the standard dependence of pressure on  $\Theta$ . In this case it is convenient to replace the vertical coordinate  $\Theta$  by its function  $\chi(\Theta) = -p^*(\Theta)/g$ .

As a result we come to the equation

$$\frac{d\chi}{dt} (\bar{\Omega}^{-1}) \approx - \frac{\partial}{\partial \chi} (\bar{\Omega}^{-1} \dot{\chi}). \quad (11)$$

Integrating this equation with respect to  $\chi$  from  $\chi_0 = \chi(\Theta_0)$  to 0, assuming in this case that  $\dot{\chi}(0) = 0$ , i.e., there are no heat influxes at the upper boundary of the atmosphere, we come to the working formula

$$\dot{\chi} \approx \bar{\Omega} \int_{\chi_0}^0 \frac{d\chi}{dt} (\bar{\Omega}^{-1}) d\chi$$

or, equivalently,

$$\dot{\Theta} \Big|_{\Theta=\Theta_0} = - \frac{\bar{\Omega}}{d p^*/d\Theta} \Big|_{\Theta=\Theta_0} \int_{\Theta_0}^{\Theta} \bar{\Omega}^{-2}(x, y, \Theta) \frac{d_0 \bar{\Omega}}{dt} \frac{d p^*}{d\Theta} d\Theta. \quad (12)$$

To calculate the fields of invariants  $\bar{\Omega}$  and  $\Theta$  we used FGGE level IIIA data on the zonal ( $u$ ) and meridional ( $v$ ) components of wind speed and air temperature  $T$  at 12 isobaric surfaces (1000, 850, 700, 500, 400, 300, 250, 200, 150, 100, 70, and 50 mbar) for the Northern Hemisphere. The calculation procedure is described in detail in [5,8].

We now move to a description of the procedure for calculating the field of heat influxes according to formula (12). The operator  $d_0/dt$  in it was calculated according to the following plan. The convective derivatives were calculated with the aid of central differences

$$\left( u \frac{\partial \bar{\Omega}}{a \cos \phi \partial \lambda} \right)_{i,j} \approx u_{i,j} \frac{\bar{\Omega}_{i+1,j}^n - \bar{\Omega}_{i-1,j}^n}{2a \Delta \lambda \cos \phi_i},$$

$$\left( v \frac{\partial \bar{\Omega}}{a \partial \phi} \right)_{i,j} \approx v_{i,j} \frac{\bar{\Omega}_{i,j+1}^n - \bar{\Omega}_{i,j-1}^n}{2a \Delta \phi},$$

where  $a$  is the Earth's radius,  $\phi$  is latitude,  $\lambda$  is longitude, and the subscripts  $i$  and  $j$  designate points of the spatial grid (they correspond to a geographic grid with spacing of  $2.5 \times 2.5^\circ$ ), and the superscript  $n$  designates a discrete moment of time  $\tau = n\Delta t$ , where  $\Delta t = 12$  hr, with consideration that the FGGE level IIIA archive contains material with given time spacing. To calculate the local derivative with respect to time  $\partial \bar{\Omega} / \partial t$  we used the interpolation formula

$$\left( \frac{\partial \bar{\Omega}}{\partial t} \right)_{i,j} \approx \frac{0.75 \bar{\Omega}_{i,j}^{n+1} - 0.5 \bar{\Omega}_{i,j}^n - 0.25 \bar{\Omega}_{i,j}^{n-1}}{\Delta t}.$$

This formula, which is a partial case of the Lagrange interpolation formula and requires knowledge of the fields during three successive observation periods, lets us decrease the effective time step by a factor of 2 compared to the usual formula, which uses forward-directed differences.

Despite the fact that the problem we are solving is diagnostic, i.e., does not require integration with respect to time, the calculation of the field of the derivative  $d_0 \bar{\Omega} / dt$  puts a limit on the ratio between the time and spatial steps. The spatial step should be such that during the interval between observation periods the air mass

transported with typical velocity  $V = \max\{u, v\}$  did not succeed in being transported for distances exceeding the selected spatial step. In other words, we should fulfill the required Courant-Friedrichs-Levi condition

$$\frac{V \Delta t}{\min \{a \cos \varphi \Delta \lambda, a \Delta \varphi\}} \leq 1.$$

Taking a value of 20 m/sec = 72 km/hr as  $V$ , we obtain (with consideration of the indicated effective decrease in the time step by a factor of 2)

$$\min \{a \cos \varphi \Delta \lambda, a \Delta \varphi\} \geq 72 \text{ km/hr} \cdot 6 \text{ hr} = 432 \text{ km},$$

which is almost twice the value of the spatial step of the grid which we used.

Because of this we applied the procedure of "sliding" smoothing of the fields of potential vortex, which allowed preservation of the number of grid points at which these fields are specified (having somewhat narrowed the latitude belt), but in this case it allowed an increase in the spatial step to  $10 \times 10^6$ , i.e., by 4 times. We used a nine-point smoothing scheme

$$\begin{aligned} (\bar{\Omega}_{i,j})^* = & (\bar{\Omega}_{i-4, j+4} + 2\bar{\Omega}_{i, j+4} + \bar{\Omega}_{i+4, j+4} + \\ & + 2\bar{\Omega}_{i-4, j} + 4\bar{\Omega}_{i, j} + 2\bar{\Omega}_{i+4, j} + \\ & + \bar{\Omega}_{i-4, j-4} + 2\bar{\Omega}_{i, j-4} + \bar{\Omega}_{i+4, j-4})/16. \end{aligned}$$

We calculated the heat influxes for the isentropic surface  $\Theta_0 = 307$  K. This surface is located in the troposphere and extends from tropical latitudes (where it is found approximately at the level of 700-800 mbar) to polar latitudes (where it is raised to the level of 250 mbar). The fields of heat influxes at this surface is conveniently compared with the behavior of the "main invariant pipe" (MIP), formed by the intersection of the surfaces of constant potential vortex  $\bar{\Omega}_1 = 1 \cdot 10^{-4} \text{ sec}^{-1}$  and  $\bar{\Omega}_2 = 2 \cdot 10^{-4} \text{ sec}^{-1}$  and isentropic surfaces  $\Theta_1 = 305$  K and  $\Theta_2 = 310$  K, the temporal evolution of which in the FGGE period was analyzed in detail in [8].

The integral in (12) was calculated by the formula of trapezoids, where the spacing with respect to  $\Theta$  was selected as equal to 10 K. Selected as the upper limit of integration was the surface  $\Theta_\infty = 447$  K, located near the isobaric surface of 50 mbar, at which the conditions of the absence of heat influxes was placed. To check the accuracy of integration we carried out test calculation using the Simpson formula with spacing  $\Delta\Theta = 5$  K. It turned out that our calculations coincide with the test with an accuracy of 10-15%, which can be considered quite satisfactory.

4. For processing the procedure we selected the period from 25 through 31 December 1978, during which we observed intensive blocking in the area of the North Atlantic, previously studied in [1,4]. During this week we used the described procedure to calculate the diurnal fields of  $\dot{\Theta}$  in the latitude belt of  $50^\circ\text{N} \leq \varphi \leq 77.5^\circ\text{N}$  for the time of 00:00 GMT. On the indicated maps we identified foci of heat influxes where  $\dot{\Theta} \geq +10$  K/day, and foci of heat outfluxes where  $\dot{\Theta} \leq -10$  K/day. Then the boundaries of the foci were reduced to a single geographic base (see Fig. 2). As expected (see Fig. 3a), in the indicated latitude belt of the Northern Hemisphere in winter the regions of cooling occupy a much greater area than regions of heating. Therefore the latter deserve great attention, and we will discuss their analysis in greater detail. The calculated spatial and temporal mean value of the heat influx was -1.2 K/day, which agrees well with climate estimates (see [9], p. 567).

In the latitude sector of  $90^\circ\text{W} \leq \lambda \leq 90^\circ\text{E}$  we identify two heating regions fairly clearly, the first of which is found in the area of Iceland and is evidently related to a blocking anticyclone located in that area (see map of mean pressure during indicated time period in Fig. 3b). It is convenient to note that this heat focus is associated with a region of stable cooling located over Greenland and Newfoundland. More unexpected is the region of stable heating over the area of Western Siberia, where there is a region of increased pressure, as evident in Fig. 3b.

Similar values of heat influxes (10-15 K/day) are obtained in [12,13] for cyclones observed in April 1974-1975 in the area of  $40^\circ\text{N}$  over the US. These works show that the major contribution to the diabatic heat influx is made by the realization of the latent heat of vapor formation in medium-scale cloud accumulations. In our case, a substantial role should evidently be played by other factors as well. The analysis demonstrated the following: despite the fact that air in the blocking anticyclone has an order of magnitude more moisture content than the surrounding air (specific humidity at the surface of  $\Theta_0 = 307$  K here is  $\approx 0.3-0.4$  g/kg), this does not explain the value of the obtained heating.

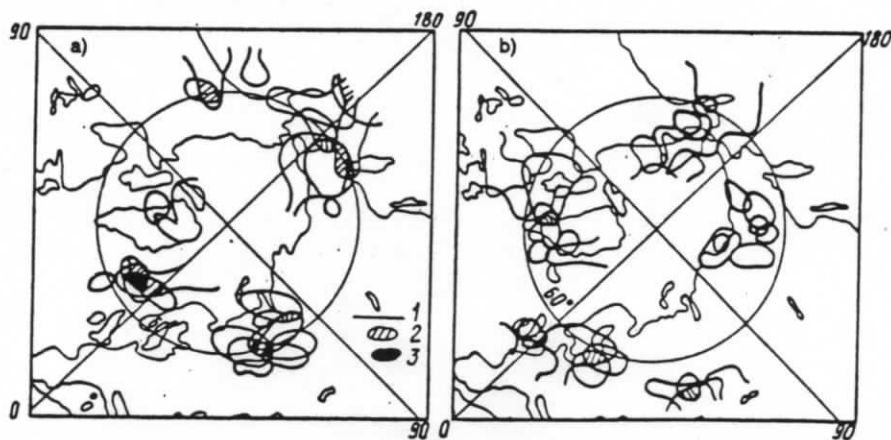


Fig. 2. Distribution of foci of heat influxes ( $\dot{\Theta} \geq 10$  K/day) (a) and heat outfluxes ( $\dot{\Theta} \leq -10$  K/day) (b) at surface of  $\Theta_0 = 307$  K during the period of 25-31 December 1978. 1) Boundaries of foci, 2) regions where foci exist during a random three days of the indicated period, 3) regions where foci exist during a random five days of the indicated period.

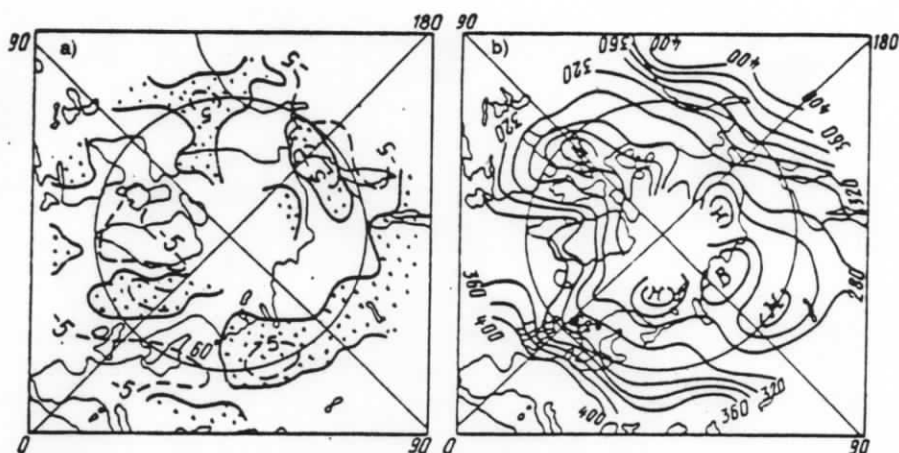


Fig. 3. Mean for period of 25-31 December 1978 at isentropic surface  $\Theta_0 = 307$  K. a) Field of heat influxes  $\dot{\Theta}$ , K/day (regions of positive heat influxes are shaded; isolines are drawn each 5 K/day); b) field of pressure  $p$ , mbar.

$$\dot{\Theta}_s = - \frac{R(\gamma_s - \gamma)}{N g} \frac{d\Theta_0}{dt} \ln \bar{\Omega}, \quad (13)$$

As evident from formula (12), the greatest contribution to the integral is made by air layers adjacent to the surface of  $\Theta_0 = 307$  K. Therefore we tried to simplify the calculation formula, having replaced the integral by the value of the subintegral function taken at the level  $\Theta_0$  and multiplied by some effective weight of the air column (smaller than actual), which can be found by correlating the calculations made by this complete and simplified formula. The latter has the form

where  $R$  is the gas constant,  $g$  is free fall acceleration,  $\gamma_s$  and  $\gamma$  are the dry-adiabatic and actual vertical gradients of temperature, respectively, and  $N > 1$  is the ratio of actual to effective weight of the air column, which is subject to definition.

Such a comparison was made for 26 December 1978 by means of constructing the regression of the field of  $\Theta_s$ , calculated by simplified formula (13), in the field of  $\dot{\Theta}$ , calculated by the complete formula (12). The regression equation has the form

$$\dot{\theta}_s = -1.08 (\pm 0.59) + 1.52 (\pm 0.07) \dot{\theta}_s$$

with the 456 taken points the correlation coefficient is 0.7. In this case the value of N turned out to be 1.5.

It is convenient to add that calculation by the simplified formula requires an order of magnitude fewer expenditures of machine time; therefore the simplified formula (13) has undoubted practical value, especially with respect to operational qualitative analysis.

Outlined above is the first experience in diagnostic calculation of the field of heat influxes, based on the equation of transformation of the potential vortex in isentropic coordinates. We obtained physically valid values of the intensity of "heat sources". The latter, moreover, have fairly distinct geographic localization, which corresponds to the circulation regime type. A future problem is the use of a larger body of data from the FGGE archive in the analysis, keeping in mind the comparison of the diagnostic calculations of "heat sources" with the types of reconstructions of circulation regimes which at a qualitative level are tracked in the field of potential vortex at isentropic surfaces.

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