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Analysis of Blocking Situations in the Atmosphere Using Adiabatic Invariants and FGGE Data

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The dynamic and statistical coupling between the geopotential field and the potential vorticity field for the blocking situation in December of 1978, characterized by a well-developed structure with splitting of the stream lines in the region of the North Atlantic, is studied. FGGE data are used. The blocking is treated as a barotropic formation. A linear dependence between the potential vorticity and the geopotential at the 500 mbar level in the blocking region is established with good statistical confidence. A diagnostic relation between dissipation and heat inflow during blocking is introduced.

INTRODUCTION

In recent years a great deal of attention has been devoted to the study of blocking situations in the atmosphere, first because of their relationship to large-scale weather anomalies and second because their study could indicate ways to improve medium- and long-range weather forecasting. The concept of blocking of the split-flow type, which is characterized by the presence of a stationary anticyclone to the north and a cyclone of approximately equal intensity to the south, in addition to which an intense jet flow is observed to the west of the blocking region, was first formulated clearly in [1]. The concepts of blocking of the meridional type and omega blocking were developed later. The key role of orography, in particular, the Rocky Mountains, in the explanation of the appearance of blocking above North America was pointed out in [2, 3]. A more detailed description of the three types of blocking indicated and their hydrodynamic interpretation are given in [4].

Study of blocking of the split-flow type is especially important for explaining large-scale weather anomalies in the territory of the USSR. The difficulty of such a study is increased by the fact that here the orography apparently does not directly affect the blocking process (as in the case of blocking of the meridional type above North America) and subtle hydrodynamic mechanisms, responsible for the formation, lifetime, and geographic correlation of the blocking structure must be sought.

Analysis of barometric charts shows that blocking structures with a characteristic lifetime of seven to ten days are observed at all basic levels from 850 to 200 and even to 100 mbar, and in addition the maxima and minima of the geopotential field clearly coincide with heating and cooling sources, respectively. This is a sufficient condition to represent the blocking formation to a first approximation as an equivalent barotropic structure, which substantially facilitates the analysis of blocking, enabling the use of the model of a barotropic atmosphere, and the blocking formation itself can be interpreted as the stationary solution of the equations of this model.

In the quasigeostrophic approximation, under adiabatic conditions (no friction and heat inflow) the law of conservation of potential vorticity for the model of a barotropic atmosphere above an orographically uneven earth can be written in the form

$$\frac{\partial}{\partial t}(\nabla^2\psi - L_0^{-2}\psi) + [\psi, \nabla^2\psi + l + L_0^{-2}\psi_g] = 0. \qquad (1)$$

Here $\psi = gz/l_0$ is the geostrophic stream function, z is the geopotential altitude, $g = 9.81$ m/sec^2 is the acceleration of gravity, $l_0 = 1.263 \times 10^{-4}$ s^{-1} is the standard value of the Coriolis parameter l evaluated at the latitude $\varphi = 60°$, $\psi_g = gh/l_0$, where h is the relief of the Earth's surface, $L_0 = c_0/l_0$ is Obukhov's scale, $c_0{}^2 = RT_0$, and T_0 is the average air temperature at the ground. The brackets denote the Jacobian. This model, as is well known, best describes the processes occurring at the middle level of the atmosphere, close to the 500 mbar isobaric surface.

In the stationary case Eq. (1) assumes the form

$$[\psi, \nabla^2\psi + l + L_0^{-2}\psi_g] = 0.$$

The necessary and sufficient condition for the Jacobian to vanish is the relation

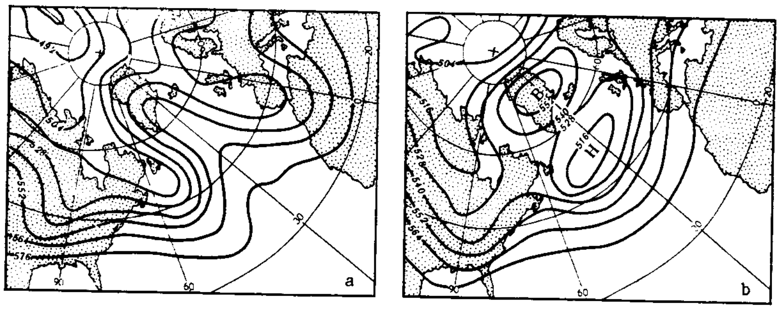

Fig. 1. Geopotential field for the isobaric surface 500 mbar: a) 0000 Greenwich Mean Time on December 19, 1978; b) at 1200 Greenwich Mean Time on December 25, 1978.

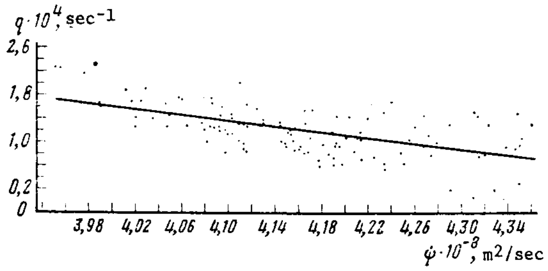

Fig. 2. Statistical relationship between the geostrophic stream function field ψ and the potential vorticity field q at 1200 Greenwich Mean Time on December 25, 1978. The equation for the regression line has the form: $q=1.23\cdot 10^{-4}-2.46\cdot 10^{-12}(\pm 0.32\cdot 10^{-12})(\psi-4.16\cdot 10^{8})$, $[q]$ = sec^{-1}, $[\psi]$ = m^2·sec^{-1}.

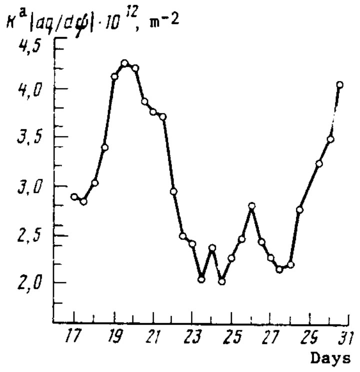

Fig. 3. The absolute values of the regression coefficient $K^2=|dq/d\psi|$ during the blocking period as a function of time.

$$q=\nabla^2\psi+l+L_0^{-2}\psi_g=\Phi(\psi),$$

where Φ is an arbitrary differentiable function.

The equations of motion of the atmosphere regarded as an ideal fluid do not permit drawing any conclusions regarding the specific form of the functional dependence $q = \Phi(\psi)$. This requires either additional physical considerations (see, for example, [5]) or a direct calculation of the function sought using real data.*

CORRELATION OF THE GEOPOTENTIAL FIELD AND THE POTENTIAL VORTICITY FIELD WITH BLOCKING

In this work the form of the function $q = \Phi(\psi)$ was determined from the meteorological information from level III-A FGGE data for the case of blocking in December of 1978. This typical split-flow blocking situation above the Atlantic was studied in [7 -9].

This blocking situation formed during December 19 - 22, when the intensification of the crest above the North Atlantic was completed by the formation of a high-pressure center in the region of Greenland with a closed 568 dam contour line on the 500 mbar surface (Fig. 1). The stage of developed blocking lasted for eight days, from December 22 to December 29. During this time the approximate coordinates of the center of the system of interacting anticyclone and cyclone were 60°N and 30°W. During the next two days the formation decayed.

The potential vorticity fields q were calculated using a spherical grid, proposed in [10]. The dependence $q(\psi)$ was determined from the values of the stream function ψ and the vorticity q from the region encompassing the blocking structure (N = 120 points). Figure 2 shows a graph of the dependence $q(\psi)$ at 1200 Greenwich Mean Time on December 25, 1978. We note that the form of the

*In laboratory experiments on the simulation of stationary vortices the functional dependence of the vorticity $\nabla^2\psi$ on the stream function ψ was calculated by Dovzhenko [6].

dependence $q(\psi)$ remains virtually unchanged as a function of time.

The dependence obtained was approximated by the linear function

$$q=q_0-K^2(\psi-\psi_0), \quad (2)$$

and the values of the parameters q_0, ψ_0, $K^2=-dq/d\psi$ were determined by the method of least squares. The quantity $K^2 = 2.46 \times 10^{-12}$ m^{-2} is essentially the square of the effective wave number of the blocking formation. Knowing the numerical value of K, it is possible to evaluate the characteristic scale of blocking using the formula $L = \pi/K \sim$ 2000 km.

A check of the statistical significance of the estimates of the correlation coefficients $r(q, \psi)$ based on a variance analysis (see, for example, [11]) indicates the significance of the values of $r(q, \psi)$, since the ratios of the variances F, owing to the regression line and the deviations from it, exceed the limiting value F = 11.38 for the limit of the confidence interval 99.9%.

We note that a linear relationship $q(\psi)$ was found in [12] based on the average monthly data for the case of an anomalous crest above the Atlantic in July of 1976 and in [13] in the numerical simulation of blocking situations. The value of K^2 found in [12] on the 300 mbar isobaric surface $K^2 = 5.25 \times 10^{-12}$ m^{-2} corresponds to $L \approx$ 1400 km.

Analysis of the changes in the coefficient of linear regression $dq/d\psi = -K^2$ as a function of the blocking time and in the period preceding blocking (Fig. 3) shows that the establishment of the blocking formation is clearly indicated by the decrease in $|dq/d\psi|=K^2$, which is linked with the presence of an anticyclonic relative vorticity ($\nabla^2\psi<0$) to the north and a cyclonic vorticity ($\nabla^2\psi>0$) to the south in the blocking structure.

DIAGNOSTICS OF NONADIABATIC FACTORS IN BLOCKING

The stationary nature of the blocking process and the existence of a functional relationship $q = \Phi(\psi)$ indicate either that the sources and sinks of potential vorticity can be neglected, i.e., blocking phenomena can be studied in the conservative approximation, or, which is more likely (since dissipative processes are effective over a characteristic time of several days), the sources and sinks q quite accurately compensate one another. In the general case of a baroclinic atmosphere for Oertel's potential vorticity

$$I=(\mathrm{rot}\,\mathbf{V}+2\Omega)\cdot\nabla s/\rho$$

such compensation conditions were formulated by A. M. Obukhov based on the equation of evolution of I [14]

$$\rho\frac{dI}{dt}=\mathrm{rot}\,\mathbf{F}\cdot\nabla s+(\mathrm{rot}\,\mathbf{V}+2\Omega)\cdot\nabla J=$$

$$\mathrm{div}\,\{\mathrm{rot}\,\mathbf{F}s+(\mathrm{rot}\,\mathbf{V}+2\Omega)J\}=0.$$

Here $\mathbf{F}$ is the force of friction, $\mathrm{rot}\,\mathbf{V}+2\Omega$ is the absolute vorticity, and $J = ds/dt$ is the intensity of the sources of entropy s.

In the case of a barotropic atmosphere the main mechanism of dissipation is Ekman friction against the Earth. The physical mechanism of friction consists of stretching of the vortex filaments in anticyclones and their compression in cyclones (we recall that the potential vorticity is the absolute vorticity per unit length of the vortex filament). The boundary layer appears to suck in air from the anticyclone and eject it upwards into the cyclone. Therefore, in order to compensate dissipation, it is necessary to introduce a distribution of heat sources and sinks on the ground which would ensure ascending motion of air at the top boundary of the boundary layer in anticyclones and, vice versa, descending motion in cyclones.

The equation of evolution of the potential vorticity in a barotropic atmosphere can be written in the form

$$\frac{\partial}{\partial t}(\nabla^2\psi-L_0^{-2}\psi)+[\psi,\nabla^2\psi+l+L_0^{-2}\psi_g]=-l_0 w/H_0, \quad (3)$$

where $H_0 = RT_0/g$ is the scale height of the atmosphere. Here w is the total vertical velocity at the top boundary of the boundary layer. It can be represented in the form of two terms: $w = w_E + w_Q$, where $w_E = \delta\nabla^2\psi$ is the vertical velocity at the top boundary of the Ekman boundary layer of thickness δ ($\sim$500 m for standard atmospheric conditions) and w_Q is the vertical velocity determined by the presence of heat sources on the surface.

We shall first study a very simple model, when the atmosphere can be represented by a layer of homogeneous incompressible liquid of height H_0, equal to the scale height of the atmosphere. We shall assume that a heat source of intensity Q_0 is concentrated in a column of liquid of height $h_0 \ll H_0$ at the bottom boundary of the atmosphere. An increase in the temperature of this column of liquid on heating by an amount ΔT increases its height by a relative amount $\Delta h/h_0=\alpha\Delta T$, where α is the coefficient of thermal expansion. In the differential form we shall have $w_Q=dh/dt=\alpha h_0 dT/dt=\alpha h_0 Q_0/c_p$. Introducing the average heat inflow per unit mass $Q = h_0 Q_0/H_0$, we arrive at the final formula

$$w_Q=\alpha H_0 Q/c_p. \quad (4)$$

Finally, in order for the functional relation $q = \Phi(\psi)$ to exist in the stationary case, it is necessary that $w_E = -w_Q$ or

$$\delta\nabla^2\psi=-\alpha H_0 Q/c_p, \quad (5)$$

which makes it possible to determine from the potential vorticity field the field of heat sources Q required for maintaining the given stationary motion.

Heat inflows can be introduced more accurately into Eq. (3) on the basis of the model proposed in [15]:

$$\frac{d\bar{u}}{dt}=-\frac{1}{\hat{\rho}}\frac{\partial\hat{p}}{\partial x}+l\bar{v},\quad \frac{d\bar{v}}{dt}=-\frac{1}{\hat{\rho}}\frac{\partial\hat{p}}{\partial y}-l\bar{u},$$

$$\frac{d\hat{\rho}}{dt}+\hat{\rho}\left(\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}\right)=0,\quad \frac{d\Theta}{dt}=\frac{Q}{c_p(1-k)}\left(\frac{\hat{\rho}_0}{\hat{\rho}}\right)^k,$$

$$\frac{1}{\Theta}\frac{d\Theta}{dt}=\frac{1}{\hat{p}}\frac{d\hat{p}}{dt}-\frac{k+1}{\hat{\rho}}\frac{d\hat{\rho}}{dt},\quad \frac{d}{dt}=\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+\bar{v}\frac{\partial}{\partial y},$$

where x and y are Cartesian coordinates, the x axis is oriented eastward, and the y axis is oriented northward. Here we introduce the following symbols indicating averaging of the meteorological characteristics over the altitude:

$$\hat{A}=\int_0^\infty A dz,\quad \bar{B}=\int_0^\infty \rho B dz/\hat{\rho}.$$

Thus $\hat{\rho}g$ is the surface pressure $p_0(x, y, t)$. It is assumed that the velocity components u and v are virtually independent of the altitude z. Here $k=R/c_p$, $\hat{\rho}_0=p_{00}/g$, $p_{00}=1000$ mbar. The potential temperature $\Theta(x, y, t)$ is constant over the height in the column of air, but can vary very slowly from one air column to another. The heat inflows $Q(x, y, t)$ are assumed to be constant as a function of altitude.* Assuming that the slow changes in $\hat{p}$ owing to the heat inflows can be neglected compared with the changes in $\hat{\rho}$, and using the equation of continuity, we can write approximately

$$\frac{1}{\Theta}\frac{d\Theta}{dt}\approx(k+1)\left(\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{v}}{\partial y}\right),$$

i.e., local heating causes the air column to expand in the horizontal direction.

Replacing in the equations of motion $\hat{\rho}$ by $\hat{\rho}_{av}$ = const, i.e., neglecting the horizontal baroclinicity (see [15]), we shall write down the equation for the curl of the velocity in which we express the horizontal divergence using the preceding formula and $d\Theta/dt$ from the equation of heat inflow. As a result, in the quasigeostrophic approximation we arrive at the equation (neglecting the effects of friction and orography)

$$\frac{d}{dt}(\nabla^2\psi+l)=-\frac{l_0}{1-k^2}\frac{Q}{c_pT_0}.$$

and thus $w_Q=QH_0/c_pT_0(1-k^2)$. For air $\alpha=1/T_0$, $k^2=4/49\ll 1$, so that the formula obtained for w_Q is virtually identical to (4) and instead of (5) we arrive at the relation

$$Q=-\frac{c_p}{R}g\delta\nabla^2\psi. \tag{6}$$

Thus, the dipole structure of blocking of the split-flow type with a warm anticyclone to the north ($\nabla^2\psi<0$) and cold cyclone to the south ($\nabla^2\psi>0$) must be maintained by heating ($Q>0$) in the region of the anticyclone and cooling ($Q<0$) in the region of the cyclone, and in addition the isolines of heat inflow Q = const must approximately coincide with the isolines of the relative vorticity.

In [9] the contribution of heat sources to generation of accessible potential energy for blocking situations in the winter of 1978 - 1979, including also for the blocking which we examined (December 19 - 29, 1978), was estimated. According to [9], a region of positive values of the covariation $\widetilde{QT}$ (the tilde sign indicates time averaging) is observed in the blocking region, which corresponds to local generation of accessible potential energy.

We calculated the covariation field $\widetilde{\nabla^2\psi T}$ from the period December 24 to December 29 (Fig. 4). It is evident that negative values of this quantity, reaching maxima (in absolute magnitude) in the region of the centers of the anticyclone and cyclone, dominate in the blocking region. Together with the results of [9], this confirms the validity of the relation (6).

The arguments presented above, which have the advantage of being convenient, can still only provide a qualitative description of compensation processes in blocking, leading to unjustifiably large values of Q, which are required for maintaining circulation, which is attributed to the low efficiency of the mechanism examined for introducing heat inflows into the barotropic model. A more realistic model can be constructed by considering the general condition for compensation of nonadiabatic factors in the stratified atmosphere

$$(\mathrm{rot}\,\mathbf{F})_z\frac{\partial s}{\partial z}+(\mathrm{rot}\,\mathbf{V}+2\Omega)_z\frac{\partial J}{\partial z}\simeq 0, \tag{7}$$

where the fact that the entropy s and its inflows J vary as a function of altitude much more than as a function of the horizontal coordinates is taken into account.

We shall determine the vertical component of the vorticity of the friction forces $(\mathrm{rot}\,\mathbf{F})_z$, assuming that the main dissipation occurs in the Ekman boundary layer, where the components of the horizontal wind velocity $\mathbf{V}=(u, v)$ are distributed as a function of altitude according to the law

*It can be shown, however, that the result is insensitive to the details of the distribution of Q as a function of altitude, if it is assumed, as is true in reality, that the maximum values of Q occur at the bottom of the atmosphere.

$$u = u_g(1 - e^{-z/h_*}\cos z/h_*) - v_g e^{-z/h_*}\sin z/h_*,$$
$$v = v_g(1 - e^{-z/h_*}\cos z/h_*) + u_g e^{-z/h_*}\sin z/h_*.$$

Here $V_g = (u_g, v_g)$ is the geostrophic wind at the top of the boundary layer. It may be assumed that $F \approx \nu_T \partial^2 v/\partial z^2$, and ν_T is the coefficient of turbulent viscosity which is related to h_* by the relation $h_*^2 = 2\nu_T/l_0$. Calculating $(\text{rot } F)_z$ based on the preceding formulas, setting in (7) $\partial s/\partial z = c_p N^2/g$, where N^2 is the squared Brunt-Väisälä frequency ($N^2 = g(\gamma_a - \gamma)/T$, $\gamma_a = g/c_p$, and $\gamma \approx {}^2/_3\gamma_a$ is the actual temperature gradient), and taking into account the quasigeostrophicity, we arrive at the equation

$$l_0\frac{\partial J}{\partial z} + \frac{c_p N^2}{g}\nabla^2\psi_0\frac{\partial \tau}{\partial z} = 0.$$

Here

$$\nabla^2\psi_0 = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y},\quad \tau = \frac{\nu_T}{h_*}\left(\sin\frac{z}{h_*} + \cos\frac{z}{h_*}\right)e^{-z/h_*}.$$

We integrate this equation over the altitude, assuming that the inflows of entropy vanish at infinity, and we find the value of these inflows J_0 at the surface required in order to compensate the effect of dissipation in the entire column of air:

$$J_0 = -\frac{c_p N^2}{g}\nabla^2\psi_0\frac{h_*}{2}. \tag{8}$$

This condition, like the one above, is written in the form $w_E + w_Q = 0$, where $w_E = \nabla^2\psi_0 h_*/2$ is the vertical velocity at the top of the Ekman boundary layer, while $w_Q = (g/c_p N^2)J_0 = J_0(\partial s/\partial z)^{-1}$ is the vertical velocity determined by the inflow of entropy (the heat inflow $Q_0 = J_0 T_0$).

For making numerical estimates we assume that $\delta = h_*/2 = 500$ m. According to our data, the relative vorticity $\nabla^2\psi$ at the 500 mbar level reaches a value of $-0.5\cdot10^{-4}$ sec^{-1} in the anticyclone. Assuming that $\nabla^2\psi_0 \approx 0.4\nabla^2\psi$ [3], we find that $Q_0 = 327$ cm$^2\cdot$sec^{-3}. Under the assumption that $h_* \ll H_0$ (H_0 is the scale height of the atmosphere), it is easily found that the average value of the heat inflows, averaged over the mass of the air column, $\bar{Q}$ is related to Q_0 by the relation $\bar{Q} \simeq Q_0 h_*/H_0$. Setting $H_0 = 8$ km and taking for Q_0 and h_* the above-indicated values, we find $\bar{Q} \simeq 41$ cm$^2\cdot$sec^{-3}, which rescaled to the air mass lying above a unit surface area, equals 41 W/m^2. We note for comparison that according to [16, p. 547], the rate at which kinetic energy is generated in separate cyclones for the four cases presented above equalled 51, 17, 21, and 16 W/m^2, respectively. At the same time, the average generation of kinetic energy northward of 32°N equals 5.3 W/m^2 [16].

Fig. 4. The covariation field $\widetilde{\nabla^2\psi T}$ (10^{-4} sec$^{-1}\cdot$K) at the 500 mbar level for the period from December 24 to December 29, 1978. The region of negative values of $\widetilde{\nabla^2\psi T}$, is marked with the single hatching, and the regions where $\widetilde{\nabla^2\psi T} < -2\cdot10^{-4}$ sec$^{-1}\cdot$K are marked with the double hatching.

It has not been excluded that the mechanism examined could be responsible for maintaining anticyclonic circulation during superprolonged summer blockings, resulting in severe droughts. In this case favorable radiation conditions are created in the anticyclone for heating the Earth's surface and the surface air layer, which according to [8], helps to maintain the anticyclone.

We note in conclusion that the calculation of the potential vorticity field, on which the analysis of blocking situations is based in this work, involves unavoidable errors in the numerical differentiation of the starting meteorological data. The accuracy of the calculation of the potential vorticity fields was questioned in [17], where, in particular, it was pointed out that a 20% relative error in 5% of the starting data on the geopotential field can lead to large (up to 100%) errors in the variance of the vorticity. However, the possibility, which has appeared since then, of employing FGGE data, subjected to careful objective analysis and represented on a grid 2.5° × 2.5° (as opposed to 5° × 10°, as used previously), enables in the opinion of the authors reliable calculations of the vorticity field. This question, however, requires a special analysis.

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